Simulation of Nonlinear Microwave Circuits — an Historical Perspective and Comparisons

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Abstract

The nonlinear analysis of microwave circuits has seen considerable development over the last decade. By assuming that only a finite number sinusoids are present in a nonlinear circuit, the computational burden of computing the transient response of the circuit is avoided and only the steady state response, given by the amplitudes and phases of the sinusoids, is required. This paper focuses on methods for computing this response. An historical perspective is presented. Quantitative comparisons of limitations, errors and dynamic ranges of the various methods are made for the simulation of single-tone and two-tone excitation of microwave amplifiers.

I. Introduction

Simulation of nonlinear microwave circuits is reaching maturity. The signals in a microwave circuit are steady-state and can be adequately described in the frequency domain by a finite number of sinusoids or by a set of phasors and their frequencies. The transient response is usually of little interest but the frequency components of the steady-state response must be known to great precision. For example it must be possible to distinguish an intermodulation product up to 140 dB below other components in a system.

This paper does not attempt to review the state-of-the-art has been addressed most recently in [1] and [2]. The aim here is to present an historical perspective of microwave circuit simulation highlighting some of the major advances. Milestones in the development of frequency domain analysis and of harmonic balance analysis of nonlinear microwave circuits are indicated in Fig. 1.

II. Harmonic Balance

The roots of the harmonic balance procedure are in Galerkin's method in which a solution is assumed, in our case a set of phasors, with unknown coefficients. Guesses of these coefficients are adjusted to minimize the error in the governing equations, usually the Kirchoff's current laws for nonlinear circuits. The method was applied to nonlinear mechanical systems and the term harmonic balance first used in 1937 by Kryloff and Bogoliuboff. The method was subsequently developed and applied to nonlinear circuits by Baily in 1960. In 1975 Nakhla and Vlach introduced partitioning of a circuit into linear and nonlinear subcircuits so that linear circuit reduction could be used to drastically simplify treatment of the linear circuit. The variables, often current phasors, describing the state of the nonlinear subcircuit are determined as the Fourier transform of the time-domain response of the nonlinear subcircuit. These are compared to the frequency-domain response of the linear circuit. This mixed time-domain/frequency-domain analysis, identified by the use of Fourier transforms, has become known as the harmonic balance (HB) method.

The first significant uses of HB in the analysis of microwave circuits was by Egami, in 1974, who used a Newton iteration procedure to minimize the HB error and by Kerr, in 1975, who used a relaxation iteration procedure. Both determined the local oscillator waveform in diode mixers. The relaxation technique was further refined by Hicks and Khan in 1982 and by Camacho-Penalosa in 1983. Continuation methods, critical to obtaining convergence at high input powers, were introduced by Filicori and Monaco in 1979. In the continuation methods the input power is ramped in steps and the results at one input power level used to determine the initial guess of the circuit state at the next higher input power level.

The first practical application of HB to multitone analysis was by Gilmore and Rosenbaum in 1984 in their modified HB method. In 1983 Rizzoli et al. introduced a state-variable approach to maintain conservation of charge among other attributes in using arbitrary models of nonlinear elements. They subsequently introduced multidimensional fast Fourier transforms (NFFTs) for multitone analysis in 1988. Multitone analysis using an aperiodic discrete Fourier transform was introduced by Chua and Ushida in 1981. This was extended to include quasi-analytic Jacobian determination and an orthogonal time point selection algorithm by Kundert et al. in 1988. The block Newton algorithm introduced by Chang et al. in 1990 was responsible for dramatic speed improvements in the HB procedure.

III. Frequency Domain Analysis

Frequency domain analysis of nonlinear microwave circuits is the logical extension of linear circuit analysis in the frequency domain. The cornerstone of nonlinear frequency domain analysis is Volterra series analysis developed in 1910. This theory was applied to nonlinear circuits in 1942 by Wiener and subsequently to transistor circuit analysis in 1967 by Narayanan, and to MESFET circuits in 1980 by Minasian.

The major disadvantage of Volterra series analysis had been the considerable algebraic manipulations required to determined the nonlinear transfer functions of anything but weakly nonlinear circuits. This restriction was removed via the significant efforts of Bedrosian and Rice in 1971, and Bussgang et al. in 1974 in the development of the method of nonlinear currents. This was applied to the analysis of arbitrarily complex microwave circuits by Crosmun and Maas in 1989 to obtain a noniterative analysis of strongly nonlinear circuits.

Frequency-domain spectral balance (FDSB) methods are very similar to HB methods. The only distinguishing feature is that the linear and nonlinear subcircuits are treated in the frequency domain and so no Fourier transformation is required. Instead the set of phasors, e.g. voltage, input

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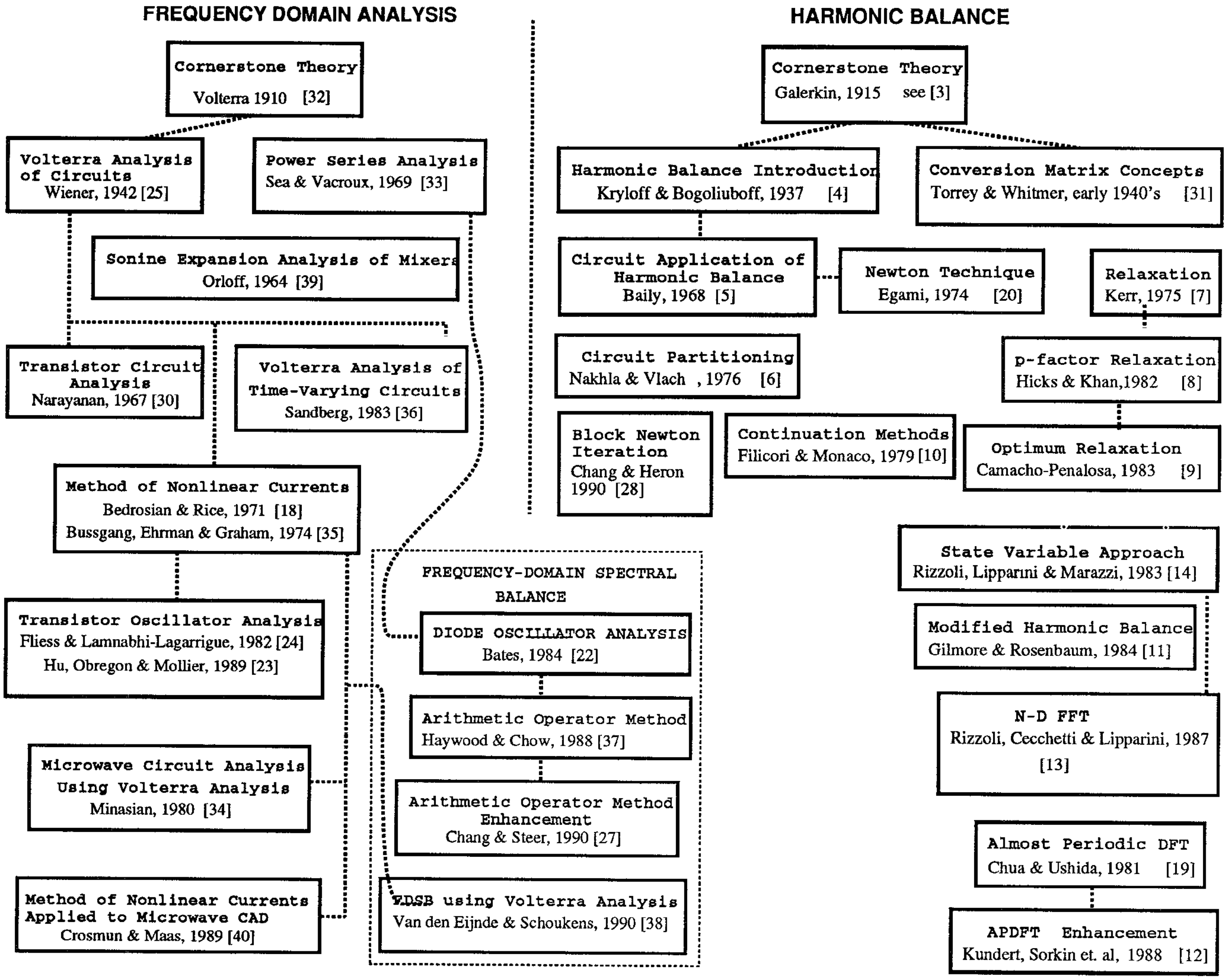

Figure 1: Historical development of nonlinear microwave circuit simulation using frequency domain analysis, frequency domain spectral balance (FDSB), and harmonic balance (HB).

to the nonlinear subcircuit is directly transformed to a set of output, e.g. current, phasors. If nonlinear elements are described by power series the input phasors map to a set of output phasors by an algebraic formula. This had been the basis for hand calculations for some time before the computer oriented technique presented by Sea and Vacroux in 1969. Subsequent developments were inefficient for handling multidimensional nonlinearities, such as a MESFET transconductance controlled by gate-source and drain-source voltages. This limitation was removed in the convolution method introduced by Haywood and Chow in 1988, and the restriction to power series descriptions removed by Chang and Steer in 1990.

IV. Transient Analysis

Transient analysis of nonlinear microwave circuits has generally been abandoned in favor of the HB and FDSB methods. Most of the efforts in transient analysis have been in the simulation of digital circuits and an historical treatment is provided in [21]. When applied to microwave circuit simulation the choice of initial conditions is particularly important in obtaining convergence. Accumulated numerical errors significantly restrict the ability to resolve small signals in the presence of large signals. Still, transient analysis of microwave circuits is the only way to observe the onset of oscillations and chaotic behavior of microwave circuits [16]. Shooting methods can be used to bypass the transient response altogether but are applicable only when the excitation is strictly periodic. In this approach the initial conditions are chosen and subsequently adjusted so that transients are not excited.

V. Comparisons

The most popular HB techniques use the APDFT or the NFFT to transform the instantaneous time-domain solutions of the nonlinear subnetwork to a set of phasors that can be used to compare the response of the nonlinear subnetwork with that of the linear subcircuit. In this section we compare these two approaches in the simulation of the MESFET amplifier shown in Fig. 2 using the techniques respectively described in [12] and [29]. Also compared in these comparisons are the simulated results of a FDSB technique using the arithmetic operator method as described in [27]. The various simulation methods were implemented in FREDA2,

[17], [27], [29], so that the only differences between the simulations were the way in which the phasors at the terminals of the nonlinear subcircuit were evaluated. In particular a dual-frequency set technique [41] was used to control the aliasing of the HB methods. Both the HB-APDFT and HB-NFFT simulations used quasi-analytic evaluation of the Jacobian.

The measured and simulated fundamental, second harmonic and third harmonic responses in a single-tone test are shown in Fig. 3. Now the APDFT reduces to a regular DFT and the NFFT to an FFT. The simulated responses using the three techniques are practically identical. Using continuation (ramping the input power in 0.1 dB steps) the amplifier response could be calculated at 50 dBm and more input power with the FDSB method. Because of the large number of harmonics that had to be simulated to avoid aliasing at higher power levels it was not practical to simulate the amplifier at these absurd power levels using the HB techniques. At 10 dBm input power the FDSB took 1.9 s, HB using NFFT took 2.5 s, and HB using APDFT took 1.7 s. The amplifier was also simulated using a general purpose commercial transient analysis program [10] but convergence could not be obtained for input powers greater than 0 dBm. Presumably the convergence properties would be improved if shooting methods were used as it would then be possible to use continuation methods to extend convergence.

The results of a two-tone test are reported in Fig. 4 and again the HB and FDSB techniques are coincident over the range shown. Meaningful simulation results could not be obtained using transient analysis. It is suspected that the accumulated numerical error in the simulated waveforms prohibited accurate Fourier transformation. Limitations of the various techniques become apparent in the two-tone results presented in Fig. 5. Here the input power of the 2.4 GHz tone (deemed the LO) is held at -5 dBm and the 2.35 GHz power (the RF) is varied. The power of the 50 MHz difference frequency (IF) is plotted. This test determines the ability to resolve a small signal, the IF, in the presence of a large signal the LO. In this case the dynamic range (the ratio of the LO to the minimum correctly resolved IF) of HB-APDFT is 80 dB and of FDSB is 470 dB. The HB-NFFT has a dynamic range of 160 dB which is also obtained using analytically determined derivatives [42]. This limitation on the dynamic range of the HB-APDFT and HB-NFFT methods is due to aliasing errors in the Fourier transformations. Numerical evaluation of the Jacobian reduces the dynamic range of the HB-NFFT method to 100 dB [42]. This limited dynamic range is due to the Jacobian not adequately representing the HB error function.

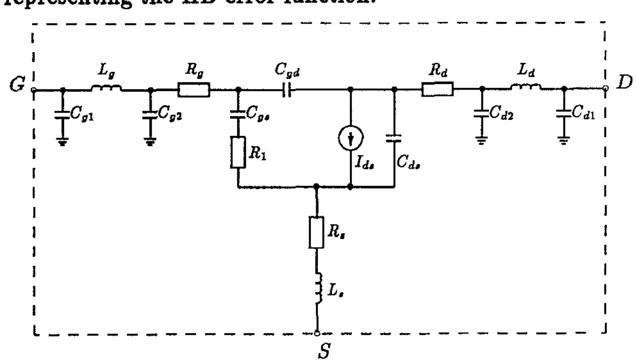

Figure 2: Circuit used to model the MESFET which includes linear as well as nonlinear elements. Nonlinear elements include C_{gs}, C_{ds}, C_{gd}, and I_{ds}, where I_{ds} is a function of both intrinsic voltages V_{gs} and V_{ds}. Element values are given in [26] and the nonlinearities are modelled by power series.

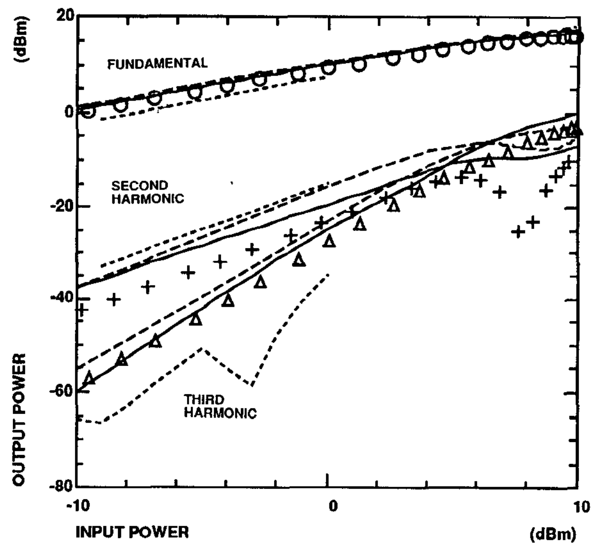

Figure 3: Results of a single-tone test Shown are the measured values (points) and the simulated results using harmonic balance (solid curves) (the results of FDSB, HB using the APDFT transform and using the NFFT transform are coincident), and the simulated results obtained using SPICE.

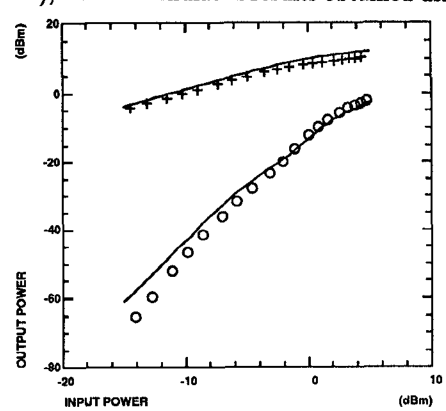

Figure 4: Results of a two-tone test plotting output power vs. power of one of the input tomes. Shown are the measured values (points) and the simulated results using harmonic balance. The input tones have equal power and are at 2.35 GHz and 2.4 GHz.

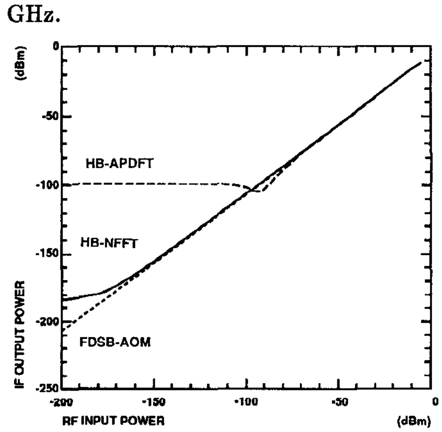

Figure 5: Comparison of simulated IF output power.

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